

A Computationally Efficient Bayesian Framework for Structural Health Monitoring using Physics-Based Models

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Abstract

A Bayesian inference framework for structural damage identification is presented. Sophisticated structural identification methods, combining vibration information from the sensor network with the theoretical information built into a high-fidelity finite element model for simulating structural behaviour, are incorporated into the system in order to monitor structural condition, track structural changes and identify the location, type and extent of the damage. The methodology for damage detection combines the information contained in a set of measurement modal data with the information provided by a family of competitive, parameterized, finite element model classes simulating plausible damage scenarios in the structure. The computational challenges encountered in Bayesian tools for structural damage identification are addressed. Simulated modal data from the Metsovo Bridge are used to validate the effectiveness of the methodology.

Keywords: Bayesian inference, structural health monitoring, damage identification, model reduction, kriging, high performance computing.

1 Introduction

Bayesian inference is used for quantifying and calibrating uncertainty models in structural dynamics based on vibration measurements, as well as propagating these uncertainties in simulations for updating robust predictions of system performance, reliability and safety [1-3]. The Bayesian tools are based on Laplace methods of asymptotic approximation [4,5] and sampling algorithms [6]. These tools involve solving optimization problems, generating samples for tracing and then populating the important uncertainty region in the parameter space, as well as evaluating integrals over high-dimensional spaces of the uncertain model and loading parameters. They require a moderate to very large number of system re-analyses to be performed over the space of uncertain parameters. For high-fidelity finite element

(FE) models required in model-based structural health monitoring applications, the use of Bayesian technique may result in excessive computations.

The computational challenges encountered in Bayesian tools for structural damage identification are addressed. An effective structural health monitoring system requires the development of computationally efficient techniques and specialized software that integrates information from physics-based mathematical models of structural components with the information collected from vibration measurements under various operational conditions, including normal operation under the action of everyday loads, wind loads and environmental effects (e.g. temperature), as well as sudden extreme load events such as moderate to strong earthquakes or strong winds. The specifications of a complete monitoring system of collecting and processing data for structural health monitoring based on physics-based FE models [7,8] are reviewed and the importance of FE model updating techniques is emphasized. Novel methods to drastically reduce computations in SHM systems are presented.

Specifically, high performance computing techniques are integrated with Bayesian techniques for structural damage identification to efficiently handle large-order models of hundreds of thousands or millions degrees of freedom, and nonlinear actions activated during system operation. Fast and accurate component mode synthesis techniques [9] are proposed, consistent with the finite element model parameterization, to achieve drastic reductions in computational effort in a single finite element model simulation. Surrogate models are also used within multi-chain MCMC algorithms with annealing properties to substantially speed-up computations [10], avoiding full system re-analyses. Significant computational savings are also achieved for highly-parallelized operations manifested in system simulations and sampling algorithms, by adopting the $\Pi 4U$ software [11] to efficiently distribute the computations in available multi-core CPUs. Such techniques allow one to handle detailed linear and nonlinear models of structural components and thus improve SHM capabilities. The importance of the proposed computational framework is demonstrated for applications on model-based structural damage identification of civil infrastructure. The developed structural health monitoring methodology is illustrated using simulated damage and corresponding modal data from a bridge of the Egnatia Odos motorway.

2 Damage identification methodology

The Bayesian inference methodology for model class selection based on measured modal data is presented in order to detect the location and severity of damage in a structure. A substructure approach is followed where it is assumed that the structure is comprised of a number of substructures and damage in the structure causes stiffness reduction in one of the substructures. In order to identify which substructure contains the damage and predict the level of damage, a family of μ model classes M_1, \dots, M_μ is introduced, and the damage identification is accomplished by associating each model class to damage contained within a

substructure. For this, each model class M_i is parameterized by a number of structural model parameters θ_i controlling the stiffness distribution in the substructure i , while all other substructures are taken to have fixed stiffness distributions equal to those corresponding to the undamaged structure. Damage in the substructure i will cause stiffness reduction which will alter the measured modal characteristics of the structure. The model class M_i that “contains” the damaged substructure i will be the most likely model class to observe the modal data since the parameter values θ_i can adjust to the modified stiffness distribution of the substructure i , while the other model classes that do not contain the substructure i will provide a poor fit to the modal data. Thus, the model class M_i can predict damage that occurs in the substructure i and provide the best fit to the data. Bayesian inference is used to identify the most probable damaged substructure given the modal data [1,12].

2.1 Bayesian model class selection

Let $D = \{\hat{\omega}_r, \hat{\phi}_r \in R^{N_0}, r = 1, \dots, m\}$ be the available measured data consisting of modal frequencies $\hat{\omega}_r$ and modeshape components $\hat{\phi}_r$ at N_0 measured DOFs, where m is the number of observed modes. Let $\Pi(\theta_i; M_i) = \{\omega_r(\theta_i; M_i), \phi_r(\theta_i; M_i) \in R^{N_0}, r = 1, \dots, m\}$ be the predictions of the modal frequencies and modeshapes from a particular model in the model class M_i corresponding to a particular value of the parameter set θ_i .

Before the selection of data, each model class M_i is assigned a probability $P(M_i)$ of being the appropriate class of models for modeling the structural behavior. Using Bayes’ theorem, the posterior probabilities $P(M_i | D)$ of the various model classes given the data D is

$$P(M_i | D) = \frac{p(D | M_i) P(M_i)}{\sum_{i=1}^m p(D | M_i) P(M_i)} \quad (1)$$

where $p(D | M_i)$ is the probability of observing the data from the model class M_i , given by

$$p(D | M_i) = \int_{\Theta_i} p(D | \theta_i, \sigma_i, M_i) \pi(\theta_i, \sigma_i | M_i) d\sigma_i d\theta_i \quad (2)$$

where $\Theta_i = \{\theta_i : \mathbf{0} < \theta_i \leq \theta_i''\}$ is the domain of integration in Equation (2) that depends on the range of variation of the parameter set θ_i , and θ_i'' are the values of

θ_i at the undamaged condition of the structure. In Equation (2), $p(D|\theta_i, \sigma_i, M_i)$ is the likelihood of observing the data from a given model in the model class M_i . For each model class M_i this likelihood is obtained using predictions $\Pi(\theta_i; M_i)$ and the associated probability models for the vector of prediction errors $e^{(i)} = [e_1^{(i)}, \dots, e_m^{(i)}]$ defined as the difference between the measured modal properties involved in D for all modes $r = 1, \dots, m$ and the corresponding modal properties predicted by a model in the model class M_i . The model error $e_r^{(i)} = [e_{\omega_r}^{(i)} \ e_{\phi_r}^{(i)}]$ for the model class M_i are assumed to be given separately for the modal frequencies and mode shapes from the prediction error equations:

$$\hat{\omega}_r = \omega_r(\theta_i; M_i) + \hat{\omega}_r e_{\omega_r}^{(i)} \quad r = 1, \dots, m \quad (3)$$

$$\hat{\phi}_r = \beta_r^{(i)} \phi_r(\theta_i; M_i) + \|\hat{\phi}_r\| e_{\phi_r}^{(i)} \quad r = 1, \dots, m \quad (4)$$

where $\beta_r^{(i)} = \hat{\phi}_r^T \phi_r^{(i)} / \phi_r^{(i)T} \phi_r^{(i)}$, with $\phi_r^{(i)} \equiv \phi_r(\theta_i; M_i)$, is a normalization constant that accounts for the different scaling between the measured and the predicted modeshape. The model prediction errors are due to modeling error and measurement noise. Assuming that they are modeled as independent Gaussian zero-mean random variables with variance σ_i^2 , the likelihood is given by [13]

$$p(D | \theta_i, \sigma_i, M_i) \sim \frac{1}{\sigma_j^{N_j/2}} \exp \left[-\frac{N_j}{2\sigma_j^2} J(\hat{\theta}_i; M_i, D) \right] \quad (5)$$

where

$$J(\theta_i; M_i, D) = \frac{1}{m} \sum_{r=1}^m \left[\frac{\omega_r(\theta_i; M_i) - \hat{\omega}_r}{\hat{\omega}_r} \right]^2 + \frac{1}{m} \sum_{r=1}^m \frac{\|\alpha_r \phi_r(\theta_i; M_i) - \hat{\phi}_r\|^2}{\|\hat{\phi}_r\|^2} \quad (6)$$

represents the measure of fit between the measured modal data and the modal data predicted by a particular model in the class M_i , $\|\cdot\|$ is the usual Euclidian norm, and $N_j = m(N_0 + 1)$.

Also, given the model class M_i , the prior probability distribution $\pi(\theta_i, \sigma_i | M_i)$, involved in Equation (2), of the model and the prediction error parameters $[\theta_i, \sigma_i]$ of the model class M_i are assumed to be independent and of the form $\pi(\theta_i, \sigma_i | M) = \pi_\theta(\theta_i) \pi_\sigma(\sigma_i)$.

The optimal model class M_{best} is selected as the model class that maximizes the probability $P(M_i | D)$ given by Equation (5). The probability distribution

$p(\boldsymbol{\theta}_i|D, M_i)$ quantifying the uncertainty in the parameters $\boldsymbol{\theta}_i$ of a model class M_i given the data is obtained by applying Bayes' theorem [1], as follows

$$p(\boldsymbol{\theta}_i, \boldsymbol{\sigma}_i|D, M_i) = \frac{p(D|\boldsymbol{\theta}_i, \boldsymbol{\sigma}_i, M_i) \pi(\boldsymbol{\theta}_i, \boldsymbol{\sigma}_i|M_i)}{p(D|M_i)} \quad (8)$$

Where $p(D|M_i)$ is the evidence of the model class M_i . The most probable value of the parameter set that corresponds to the most probable model class M_{best} is denoted by $\hat{\boldsymbol{\theta}}_{best}$.

Using the Bayesian model selection framework in Section 2, the model classes are ranked according to the posterior probabilities based on the modal data. The most probable model class M_{best} that maximizes $p(M_i|D)$ in Equation (5), through its association with a damage scenario on a specific substructure, will be indicative of the substructure that is damaged, while the most probable value $\hat{\boldsymbol{\theta}}_{best}$ of the model parameters of the corresponding most probable model class M_{best} , compared to the parameter values of the undamaged structure, will be indicative of the severity of damage in the identified damaged substructure.

The percentage change $\Delta\boldsymbol{\theta}_i$ between the best estimates of the model parameters $\hat{\boldsymbol{\theta}}_i$ of each model class and the values $\hat{\boldsymbol{\theta}}_{i,und}$ of the reference (undamaged) structure is used as a measure of the severity (magnitude) of damage computed by each model class M_i , $i = 1, \dots, \mu$.

The selection of the competitive model classes M_i , $i = 1, \dots, \mu$ depends on the type and number of alternative damage scenarios that are expected to occur or desired to be monitored in the structure. The μ model classes can be introduced by a user experienced with the type of structure monitored. The prior distribution $P(M_i)$ in Equation (5) of each model class or associated damage scenario is selected based on the previous experience for the type of bridge that is studied. For the case where no prior information is available, the prior probabilities are assumed to be equal, $P(M_i) = 1 / \mu$, for all introduced damage scenarios.

3 Computationally Efficient Techniques

The Bayesian tools for FE model selection and parameter estimation used for structural damage identification are Laplace methods of asymptotic approximation and stochastic simulation algorithms. These tools require a moderate to very large number of repeated system analyses to be performed over the space of uncertain parameters. Consequently, the computational demands depend highly on the number of system analyses and the time required for performing a system analysis. The

Transitional MCMC (TMCMC) algorithm [14] is employed in this work to carry out the computations. High performance computing techniques are integrated within the TMCMC tool to efficiently handle large number of DOF in FE models. Specifically, fast and accurate component mode synthesis techniques [9] are used, consistent with the FE model parameterization, to achieve drastic reductions in computational effort. Surrogate models [10] are also used to replace full system simulations by fast approximations. Finally computational savings are achieved by adopting parallel computing algorithms to efficiently distribute the computations in available multi-core CPU [15]. Details of such HPC techniques are presented in [11] where the software $\Pi 4U$ is introduced to perform a model non-intrusive Bayesian analysis in a parallel environment.

The model reduction techniques and the surrogate modeling based on kriging algorithm are briefly described in the following.

3.1 Model Reduction

At the model level, model reductions techniques have been proposed to considerably reduce the size of the stiffness and mass matrices by several orders of magnitude. In particular, computational efficient model reduction techniques based on component mode synthesis have been developed to handle certain parameterization schemes for which the mass and stiffness matrices of a component depend either linearly or nonlinearly on only one of the free model parameters to be updated, often encountered in FE model updating formulations. In such schemes, it has been shown that the repeated solutions of the component eigen-problems are completely avoided, reducing substantially the computational demands, without compromising the solution accuracy. For the case of linear dependence of the stiffness matrix of a structural component on a model parameter, the methodology is presented in [9]. The method can readily be extended to treat the case of nonlinear dependence of the stiffness matrix of a structural component on a model parameter.

3.2 Surrogate models

At the level of the TMCMC algorithm, an adaptive kriging model has been introduced in [10,15] to reduce the computational time by avoiding the full model runs at a large number of sampling point in the parameters space. This is done by exploiting the function evaluations that are available at the neighbour points from previous full model runs in order to generate an estimate at a new sampling point in the parameter space. Surrogate models are well-suited to be used within the TMCMC algorithm, resulting to the X-TMCMC algorithm proposed in [10]. In X-TMCMC, the kriging technique is used to approximate the function evaluation at a sampling point using the function evaluations at neighbour points in the parameter space. To ensure a high quality approximation, certain conditions are imposed in order a surrogate estimate be accepted. Details can be found in [10]. In contrast to the model reduction technique in which several orders of magnitude reduction in computational effort may be achievable, for surrogate models within TMCMC only an order of magnitude reduction in the number of full model runs involved in

TMCMC algorithm has been reported which results in additional computational savings.

4 Application

The methodology is demonstrate using a simulated damage scenario and simulated modal data from the Metsovo bridge, shown in Figure 1. The bridge is the highest reinforced concrete bridge of Egnatia Odos motorway located in Greece, with the height of the taller pier P2 equal to 110m. The total length of the bridge is 537m. The commercial software package COMSOL Multiphysics is used for developing the FE model of the bridge based on the design plans, the geometric details and the material properties of the structure. The following nominal values of the material properties of the concrete deck, piers and foundations are considered: Young's modulus $E = 37Gpa$, Poison's ratio $\nu = 0.2$ and density $\rho = 2548kg/m^3$. For the piers and the foundation the nominal value of the Young's modulus is $E = 34GPa$. A detailed FE model is created using three-dimensional tetrahedron quadratic Lagrange finite elements. The selected model has 97,636 finite elements and 562,101 DOF.



Figure 1: Metsovo bridge.

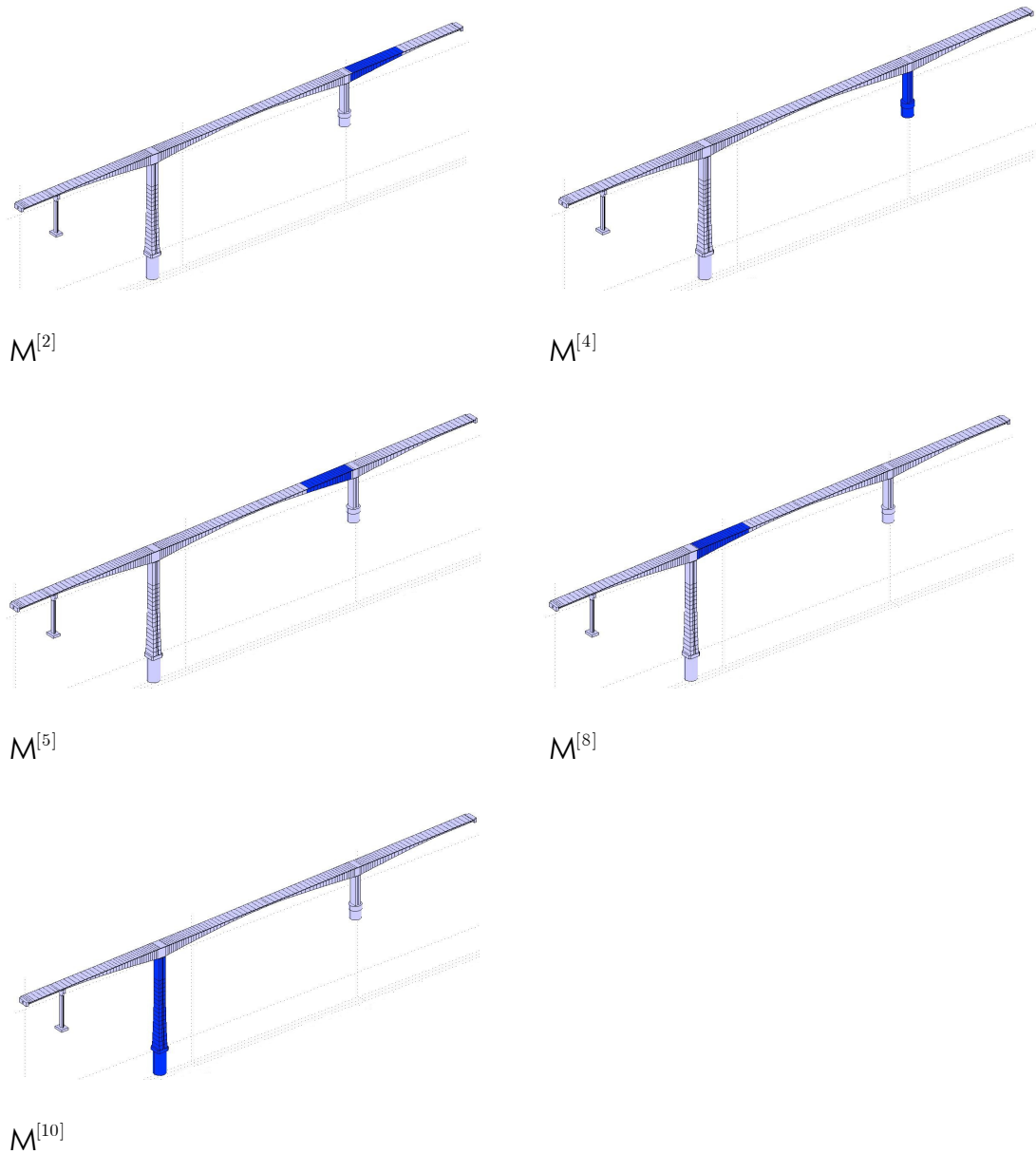


Figure 2: Parameterized FE models corresponding to single damage scenarios.

To demonstrate the methodology, the Metsovo bridge is divided into 15 substructures. A number of competitive model classes $M^{[i]}$ and $M^{[i,j]}$ are introduced to monitor various probable damage scenarios for the bridge corresponding to single and multiple damages at different substructures. The model class $M^{[i]}$ contains one parameter related to the stiffness of substructure i . Representative models corresponding to a single damage scenario at different substructures are shown in Figure 2. These models can monitor damage associated with the stiffness reduction in the i substructure. The model class $M^{[i,j]}$ contains two parameters related to the

stiffness of substructures i and j . Representative models are shown in Figure 3 and they can be used to monitor damage associated with the stiffness reduction in either substructures i and j or simultaneously at both substructures. A five-parameter model shown in Figure 3 is also included in the family of model classes to monitor simultaneous damages at five different substructures. This five-parameter model class is denoted by $M^{[5-par]}$.

All model classes are generated from the updated FE model of the undamaged structure. For each model class, CMS techniques are used to alleviate the computational burden associated with the model updating problems that needs to be solved. For this, two different cases of reduced-order FE models are considered. The first case corresponds to models obtained by reducing the internal DOF, while the second case corresponds to models obtained by reducing both the internal and interface DOF.

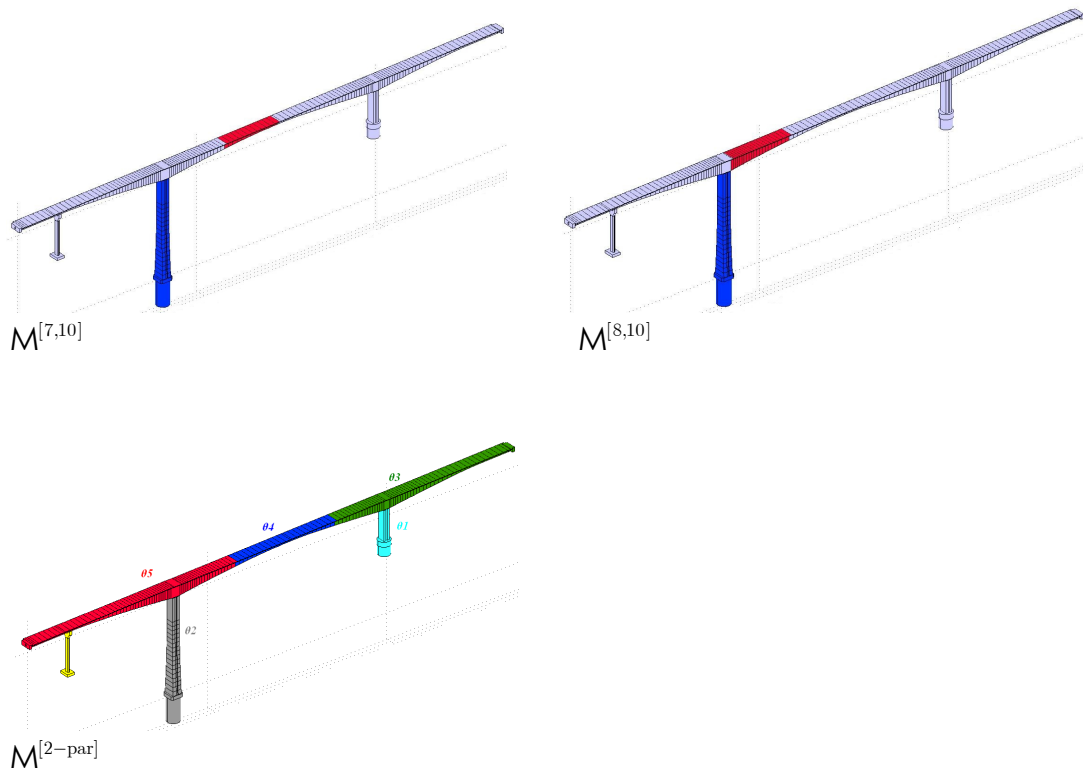


Figure 3: Parameterized FE models corresponding to multiple damage scenarios.

The number of components introduced for each model class depends on the parameterization. Specifically, the model class $M^{[i]}$ is divided into two, three or four components. One component is selected to be the substructure i shown in Figure 2, while the remaining components are selected to be the parts of the remaining structure that connect to the interfaces of component i . The model classes $M^{[1]}$, $M^{[10]}$, $M^{[14]}$ and $M^{[15]}$ have one interface, the model classes $M^{[2]}$, $M^{[5]}$ to $M^{[8]}$, $M^{[11]}$

and $M^{[12]}$ have two interfaces, while the model classes $M^{[3]}$, $M^{[9]}$ and $M^{[13]}$ have three interfaces with the remaining structure. A similar division into components is introduced for the family of $M^{[i,j]}$ model classes. For example, model class $M^{[10,8]}$ is divided into four components, the first two components coincide with the physical substructures 10 and 8, the third includes the physical substructures 9, 11 to 15 and the fourth includes the substructures 1 to 7. The components in the $M^{[5-par]}$ model class are kept the same as the ones shown in Figure 3.

Model Class	Evidence (log)	$\Delta\theta_i$ (%)	DOF (NFES)	CE (Min)
$M^{[2]}$	954.46	+27.9	1,724 (8,000)	123
$M^{[4]}$	954.99	-15.7	989 (8,000)	42
$M^{[5]}$	988.17	-47.8	1,747 (9,000)	134
$M^{[8]}$	1005.5	-31.3	1,824 (9,000)	170
$M^{[10]}$	1723.1	-29.2	1,393 (12,000)	173
$M^{[10,7]}$	1722.5	-29.0 +4.0	1,829 (14,000)	245
$M^{[10,8]}$	1718.7	-29.0 +1.9	2,485 (14,000)	509
$M^{[5-par]}$	1700.4	-1.3 -28.3 +1.0 +2.3 +0.5	3,586 (19,000)	759
Total				2155

Table 1: Damage identification results, model DOF, number of FE simulations (NFES) and computational effort (CE) in minutes for each model classes A.

The effectiveness of the proposed methodology, in terms of computational efficiency and accuracy, is investigated by introducing a simulated damage at the highest pier. The inflicted damage corresponds to a stiffness reduction of 30% the nominal stiffness value. Simulated, noise contaminated, measured modal frequencies and mode shapes are generated for the damaged structure. Among all models in Figure 2 and 3, $M^{[10]}$, $M^{[10,i]}$ and $M^{[5-par]}$ contain the actual damage.

The model class selection and the model updating is performed using the TMCMC algorithm with 1000 samples per TMCMC stage. The results for the log evidence for representative model classes and the corresponding magnitude of

damages $\Delta\theta_i$ predicted by each model class are reported in Tables 1 and 2 for the two cases A and B of reduced-order models. Herein, for demonstration purposes, the percentage change $\Delta\theta_i$ between the mean estimates of the model parameters of each model class and the corresponding values of the reference (undamaged) structure measures the severity (magnitude) of damage computed by each model class.

Model Class	Evidence (log)	$\Delta\theta_i$ (%)	DOF (NFES)	CE (Min)
$\mathcal{M}^{[2]}$	954.93	+26.5	438 (8,000)	3.5
$\mathcal{M}^{[4]}$	955.08	-15.2	381 (8,000)	3
$\mathcal{M}^{[5]}$	989.32	-47.3	441 (9,000)	3.6
$\mathcal{M}^{[8]}$	1006.4	-30.8	408 (9,000)	0.5
$\mathcal{M}^{[10]}$	1723.3	-29.2	388 (12,000)	4.6
$\mathcal{M}^{[10,7]}$	1723.1	-29.0 +3.9	425 (13,000)	5.4
$\mathcal{M}^{[10,8]}$	1719.0	-29.0 +1.3	433 (13,000)	5.5
$\mathcal{M}^{[5-par]}$	1698.2	-0.5 -28.5 +0.9 +1.5 +0.5	592 (19,000)	14
Total				40.1

Table 2: Damage identification results, model DOF, number of FE simulations (NFES) and computational effort (CE) in minutes for model classes B.

Comparing the log evidence of each model class and also the corresponding magnitude of damages $\Delta\theta_i$ predicted by each model class in Table 1 it is evident that the proposed methodology correctly predicts the location and magnitude of damage using the reduced-order model classes. Specifically, based on the reduced-order models A, the most probable model class is $\mathcal{M}^{[10]}$ which predicts a mean 29.2% reduction in stiffness which is very close to the inflicted 30%. Among all alternative model classes $\mathcal{M}^{[10]}$, $\mathcal{M}^{[10,7]}$, $\mathcal{M}^{[10,8]}$ and $\mathcal{M}^{[5-par]}$ that contain the actual damage, the proposed methodology favors the model class $\mathcal{M}^{[10]}$ with the least number of parameters and it predicts the five parameter model class $\mathcal{M}^{[5-par]}$ as the least probable model. This is consistent with theoretical results for model class penalization for over parameterization, available for Bayesian model class selection

[5]. The model classes that do not contain the damage are not favored by the proposed methodology. Based on the reduced-order models B in Table 2, the predictions of the location and severity of damage are very close to the ones obtained from the reduced-order models A for most model classes included in Table 1. In particular, the most probable model class for models B is also predicted to be $M^{[10]}$, while the mean damage severity is predicted to correspond to 29.2% reduction in stiffness, exactly the same as the one predicted with the reduced-order models A. In addition the use of kriging within TMCMC (algorithm K-TMCMC) also provides accurate estimates of the evidence that lead to damage identification results that are identical to the ones obtained without the kriging estimates.

The resulting number of FE model re-analyses and the computational demands in minutes for each model class are also shown in Tables 1 and 2. The number of FE model runs for each model class depends on the number of TMCMC stages which vary for each model class from 8 for the one-parameter model class to 19 for the five-parameter model class. The resulting variable number of stages per model class was automatically obtained from the TMCMC algorithm by keeping constant the value tolCov of the TMCMC parameter to $\text{tolCov} = 1.0$.

The parallelization features of TMCMC [11] were also exploited, taking advantage of the available four-core multi-threaded computer unit to simultaneously run eight TMCMC samples in parallel. For comparison purposes, the computational effort for solving the eigenvalue problem of the original unreduced FE model is approximately 139 seconds. Multiplying this by the number of TMCMC samples shown in Tables 1 and 2 and considering parallel implementation in a four-core multi-threaded computer unit, the total computational effort for each model class is expected to be of the order of 3 to 7 days for 8,000 to 19,000 samples, respectively.

For all eight model classes considered in Tables 1 and 2, the total computational effort using the unreduced models is estimated to be approximately one month and seven days. In contrast, for the reduced-order models A, the computational demands for running all model classes are reduced to 30 hours (2155 minutes as shown in the last row of Table 1), while for the reduced-order models B these computational demands are drastically reduced to 40 minutes (see Table 2). The use of surrogate models such as K-TMCMC [10] reduces the computational effort by an additional 85% making the total computational effort equal to approximately 6 minutes.

It is thus evident that a drastic reduction in computational effort for performing the structural identification based on a set of monitoring data is achieved from approximately 37 days for the unreduced model classes to 40 minutes for the reduced model classes B, without compromising the predictive capabilities of the proposed damage identification methodology. This results in a drastic reduction in the computational effort of more than three orders of magnitude. Additional substantial reductions in computational effort are possible if one has available more computer workers to efficiently distribute the MCMC samples in each TMCMC stage, as well as to run the different model classes, corresponding to different possible damage scenarios, in parallel. The availability of such computer workers is expected to yield the damage identification results in a few seconds, provided that a database of reduced parameterized finite element models that cover all damage scenarios have been introduced.

6 Conclusions

A structural identification framework based on vibration measurements was outlined in this work. The framework integrates Bayesian computing tools with high fidelity finite element models of the monitoring structure. It requires a large number of finite element model analyses that can result in excessive computational effort. HPC techniques were proposed to drastically reduce the computational burden by several orders of magnitude. The effectiveness of the algorithms, in terms of computational efficiency and accuracy was demonstrated using simulated modal data from the Metsovo Bridge of the Egnatia Odos motorway in Greece. The proposed framework can be used by managing authorities as part of an intelligent structural management system to provide a useful tool for structural monitoring, structural integrity assessment and design cost-effective maintenance strategies.

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